occur during the AFE flight and that adequate thermal protection should be provided.

Acknowledgments

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Startup Hydraulic Transients in Liquid-Propellant-Delivery Systems

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Nomenclature

A = cross-sectional area of pipe for fluid transport, m^2
A_a = cross-sectional area of pipe annulus, m^2
c = acoustic speed of fluid, m/s
E = bulk modulus of fluid, Pa
E_s = bulk modulus of pipe material, Pa
e = thickness of the pipe wall, m
F = magnitude of forces exerted by fluid on pipe, N
h = depth of liquid in tank, m
k = a length of pipe, m
L = a length, m
ℓ = a length of pipe, m
M = magnitude of bending moment, N · m
M_c = a momentum flux, N
p = fluid pressure, Pa
p_o = tank pressure, Pa
r = radius of a semicircle defined by pipe centerline, m
s = distance along the pipe centerline, m
t = time, s
v = speed of fluid stream when it reaches the cap, m/s
w = speed of the cap, m/s
y = a length, m
θ = cylindrical polar angle, rad
ρ = density of fluid, kg/m^3
ρ_s = density of the pipe material, kg/m^3
σ = stress, Pa

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Pipe Filling

LIQUID is delivered at high speed (~50 m/s) from a tank at pressure p_o (~10 MPa) into an initially evacuated pipe, of uniform cross-sectional area A (~0.5 cm^2) for liquid transport (Fig. 1), upon disk bursting at time $t = 0$. Breakage of the pipe may occur at the weld in the roughly U-shaped, horizontal section, which connects the short vertical section from the pressurized feed tank and the vertical-rise section leading to the cap at the dead end. We undertake a transient one-dimensional analysis of a simpler, semicircular configuration, for which the pipe centerline lies entirely in one plane (Fig. 2).

Consider the force balance along the axis of symmetry of the pipe (Fig. 2); the force of the pipe on the fluid equals the mass times the (cylindrically) radial acceleration of the fluid. At time t, the front of the liquid stream is at position $s(t)$, where the coordinate s is measured along the pipe centerline from position B; when the front is at position C, $s = \ell$. When the front is at position E, $s = \ell + r\theta$, where r is the radius of the semicircle defined by the pipe centerline, and θ is an angle

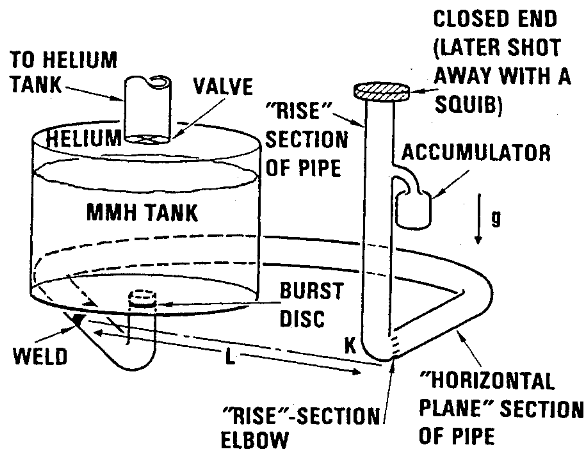

Fig. 1. Schematic of the liquid-fuel-delivery system.

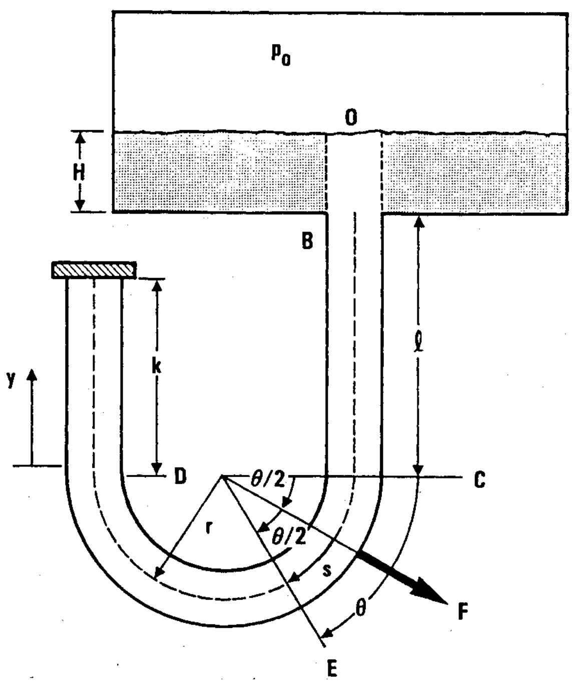

Fig. 2. Geometry analyzed for the transient hydraulic phenomena.

measured from an initial ray to position C. When the front is at y, $s = \ell + \pi r + y$. Since friction reduces the threat to the weld, we consider the ideal-fluid "worst" case.

Some fluid in the tank, along with the fluid in the pipe, is accelerated by the pressure difference $\Delta p(\approx p_o)$ across the liquid surface to the front. We take the momentum flux supplied to the pipe-entering tank fluid to be equivalent to that of a column of fluid of cross-sectional area A and height h ($\approx H$, the depth of the liquid in the pressurized feed tank).

If ρ is the fluid density, applying Newton's law gives

$$\frac{\mathrm{d}}{\mathrm{d}t}[(h+s)\rho A\dot{s}] \simeq p_o A \tag{1}$$

For $s = \dot{s} = 0$ at $t = 0$,

$$\dot{s} = \left(\frac{p_o}{\rho}\right)^{1/2} \frac{[(h+s)^2 - h^2]^{1/2}}{h+s} \tag{2}$$

When the front is at position E, the momentum flux (downward) into C at that time is

$$M_c = p_o A \frac{(h+s)^2 - h^2}{h+s}, \qquad s = \ell + r\theta \tag{3}$$

The magnitude of the momentum flux is constant with position along the length of the pipe filled with fluid, and this magnitude varies in time with fluid-front position. Thus, the momentum flux through E is of magnitude M_c (but is directed at an angle θ to the vertical), at the same time that the momentum into C is M_c (and directed downward). The difference in these two fluxes is the (resultant) force F exerted by the fluid on the tube, where the magnitude F of the force is

$$F = 2M_c \sin(\theta/2) \tag{4}$$

Although the weld is not quite at position C, the moment arm applicable to the moment of the force F about the position C is $[r \sin(\theta/2)]$, and the magnitude of the bending moment M is

$$M = 2M_c r \sin^2(\theta/2) \tag{5}$$

The bending moment is such that the "outside" of the pipe is in compression and the "inside" is in tension.

When the fluid front is at D, Eq. (5) still applies, and $M = 2M_c r$, where $s = s_D \equiv \ell + r\pi$; this value of M continues to hold when the fluid front is in the rise section of the pipe ($s > s_D$), but the value of s appropriate to the position y is $s = \ell + \pi r + y$, as already noted.

The moment first appears when the liquid front reaches C, increases monotonically from zero until the time at which the fluid front reaches the exit of the semicircular section of Fig. 2 (or the U-shaped section in Fig. 1), and holds at the maximum value until the pressure p_o falls off or fluid reaches the cap at the end of the rise. If the radius of the semicircular section is $\mathcal{O}(10$ cm), the maximum moment is several thousands of Newton-centimeters. Reducing the pressure p_o would clearly help, but clamping the rise section of the pipe and adding an accumulator (Fig. 1) afford little assistance.

Waves Initiated When the Fluid Front Reaches the Cap

When the fluid hits the dead end, a shock wave is reflected in the fluid back down the rise section of the pipe. Simultaneously, the end plate exerts an upward force on the metal annulus that is at the top of the pipe. A tensile stress wave, whose bottom edge travels at the acoustic speed of the metal, $(E_s/\rho_s)^{1/2}$, propagates in the pipe. The tensile stress σ in the pipe is given by

$$\sigma = \rho_s (E_s/\rho_s)^{1/2} w \tag{6}$$

where w is the speed of the cap and all of the pipe down as far as the position $(E_s/\rho_s)^{1/2}\, t$, where t is now defined to be the time since the fluid front arrives at the cap. The pressure in the fluid is

$$p = \rho c(v - w) \tag{7}$$

where the acoustic speed in the fluid c is given by, for a thin-wall circular pipe,[1]

$$c \simeq \frac{(E/\rho)^{1/2}}{[1 + (E/E_s)(d/e)]^{1/2}} \tag{8}$$

with e denoting the thickness of the pipe wall, and E the fluid bulk modulus. All of this treats the cap as massless and rigid.

The dynamic equilibrium of the cap requires that

$$pA = \sigma A_a \tag{9}$$

Thus,

$$\rho c(v - w)A = (E_s \rho_s)^{1/2} w A_a \tag{10}$$

This relation gives w, and σ follows from Eq. (6) and p from Eq. (7).

Since in steel $(E_s/\rho_s)^{1/2} \simeq 5100$ m/s and in water $c \simeq 1500$ m/s, the stress wave in the pipe propagates faster. When the wave reaches position K at the bottom of the rise section (Fig. 1), the vertical force exerted on the end of the U-shaped loop implies a flexure of the pipe and (when the wave arrives at the weld) causes a tensile stress at the bottom of the weld and compressive stress at the top, plus maybe some twist.

When the pressure wave in the liquid gets to the elbow K at the bottom of the rise section, there is a further reflected shock (acoustic wave) up the rise section and a transmitted wave into the U-shaped, horizontal loop. The pressure, higher than the value p computed via Eq. (7), would be hard to calculate. However, the new pressure p_2 in the elbow at position K and behind the wave fronts emanating from the elbow must be between the values p and $2p$. The (horizontal) force exerted on the vertical pipe by the pressure wave is zero on any ring that lies above the elbow, but down in the elbow the projected area (denoted by a vertical row of marks near position K in Fig. 1) is not pipe; this area has cross section A, and therefore the magnitude of the net force of liquid on the elbow is $p_2 A$. This force also induces a flexure in the loop and, when the bending wave so initiated reaches the vicinity of the weld, the bending moment is the vector (cross) product of the force whose magnitude is $p_2 A$ and the length L (Fig. 1). The stresses that are induced in the weld section and that constitute the bending moment in the horizontal plane are compressive on the inside of the loop and tensile on the outside. The time of arrival at the weld of the bending wave, and the duration of the bending, are difficult to quantify. However, the bending of a straight beam that flexes in the vertical is described by

$$E_s I_s u_{xxxx} + A_s \rho_s u_{tt} = 0 \tag{11}$$

where u is vertical displacement (so that u_{xx} is the curvature). The spread of a disturbance is characterized by the quantity $\kappa = [(E_s I_s)/(A_s \rho_s)]^{1/2}$, where I_s is the cross-sectional moment of inertia of the annulus. [I_s has units of (length)^4 and κ has units of a diffusion coefficient.] The arrival time is $\mathcal{O}(L^2/\kappa)$, where L is the pertinent length scale. This excitation (the stress-bending wave induced when the vertical pressure pulse hits the elbow at position K) seems the only viable candidate for the failure at the weld if the failure is associated with compression on the inside of the loop and tension on the outside. We anticipate that an accumulator can alleviate, not eliminate, this threat of failure.

Anchoring so that the elbow is horizontally immovable would completely eliminate the excitation just described, for then all of the force would push on the anchor. If clamping the elbow makes no difference to weld survival, then plausibly the principal threat to the weld arises during pipe filling, not from waves generated by fluid interaction with the cap.

Acknowledgment

I gratefully thank George Carrier for helpful discussions.

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Errata

Proposed Radiometric Measurement of the Wake of a Blunt Aerobrake

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IN the original publication, the names of Drs. Venkatapathy and Craig were omitted from the author list owing to a NASA Ames management oversight.

W. C. Davy and R. A. Craig originally proposed and led the development of the Afterbody Radiometer Experiment from 1986–1989 and the motivation for the paper is based on this contribution. Dr. Venkatapathy led the CFD group effort in computing the base flow properties of the spacecraft. This was critical activity for the instrument definition and was the theoretical basis for the paper.

Drs. Venkatapathy and Craig had principal roles in the subject activity of the paper and they deserve recognition for their contributions as do the previously listed authors.

Submitted by Dr. James O. Arnold, Chief, Thermosciences Division, NASA Ames Research Center.